

Godfrey Harold Hardy (1877–1947) was the most important British pure mathematician of the first half of the 20th century. Although he is usually thought of as a Cambridge man, his years from 1920 to 1931 as Savilian Professor of Geometry at Oxford University were actually his happiest and most productive. At Oxford he was at the prime of his creative life, and wrote over 100 papers there, including many of his most important investigations with his long-term Cambridge collaborator J E Littlewood.



Mathematical
Institute

G H Hardy's Oxford years

“I was at my best at a little past forty, when I was a professor at Oxford.”

G H Hardy
A Mathematician's Apology

“...the happiest time of his life...”

C P Snow
*Introduction to
A Mathematician's Apology*

“He preferred the Oxford atmosphere and said that they took him seriously, unlike Cambridge.”

J E Littlewood
Littlewood's Miscellany



Hardy's Oxford time-line

- 1919** Hardy appointed Savilian Professor of Geometry
- 1920** Comes into office on 19 January
18 May: Gives inaugural lecture on problems in the theory of numbers
Srinivasa Ramanujan dies
Awarded the Royal Medal of the Royal Society
- 1921** Visits colleagues in Scandinavia and Germany
- 1922** President of the British Association, Mathematics & Physics section
Presidential lecture: *The theory of numbers*
- 1924** **Georg Pólya** (first Rockefeller Fellow) visits Oxford
- 1924–26** President of the National Union of Scientific Workers
- 1925–26** **Abram Besicovitch** visits Oxford
- 1925–27** President of the Mathematical Association
Presidential lectures: *What is geometry?* and *The case against the Mathematical Tripos*
- 1926** Advocates founding the *Journal of the London Mathematical Society*
- 1926–28** President of the London Mathematical Society
Presidential lecture: *Prolegomena to a chapter on inequalities*
- 1927** Co-edits Ramanujan's *Collected Papers*
- 1928** Attends International Congress of Mathematicians in Bologna
- 1928–29** Spends the academic year in the USA (mainly at Princeton and Caltech)
Replaced in Oxford by **Oswald Veblen**
- 1929** Awarded the London Mathematical Society's De Morgan Medal
- 1930** Re-launches the *Quarterly Journal of Mathematics*
Lobbies in the *Oxford Magazine* for a Mathematical Institute
- 1931** **Albert Einstein** receives Oxford honorary degree and gives lectures
Hardy leaves Oxford and returns to Cambridge

G H Hardy's Oxford years

Savilian Professor



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The Savilian Professorships of Geometry and Astronomy at Oxford University were founded by Sir Henry Savile, Warden of Merton College, in 1619.

From Cambridge to Oxford

From 1897 to 1916 the Geometry chair was occupied by William Esson, following the tenure of J J Sylvester who was appointed to it at the age of 69 after returning from the USA.



William Esson

Esson had been largely responsible for initiating Oxford's inter-collegiate lecture system, replacing the arrangement in which the colleges mainly operated independently. He was Sylvester's deputy from 1894 to 1897 when failing eyesight prevented Sylvester from lecturing. Esson died in 1916, during World War I, and the election to the vacant Savilian Chair was suspended until the War was over.

Eventually, in October 1919, an advertisement appeared in the *Oxford University Gazette* inviting applications. The details of the post included the following:
The Professor will be a Fellow of New College, and will receive a stipend of £900 per year ... It will be the duty of the Professor to lecture and give instruction in Pure and Analytical Geometry ... He is also bound to give not less than forty-two lectures in the course of the academical year.

Although Hardy was not a geometer, a number of considerations made the Oxford position attractive to him.

By 1919 he had become disillusioned with Cambridge, where he had resided since entering as a scholar in 1896. He was sickened by the War and by the pro-War attitudes of his colleagues at Trinity College, especially in their behaviour towards Bertrand Russell, then a young lecturer.



Bertrand Russell

In 1916 Russell was convicted, and later imprisoned, for his anti-War activities. The College removed Russell's lectureship, reinstating it in 1919, but as Russell's most ardent supporter in the College, Hardy found the atmosphere stifling and needed to get away.

Hardy was also devastated by the illness and departure of his co-worker Srinivasa Ramanujan, his researches with J E Littlewood were not going well, and he was suffering from an increased administrative load.

A complete break was called for...

Hardy is appointed

In December 1919 the following notice appeared in the *Oxford University Gazette*:

SAVILIAN PROFESSORSHIP OF GEOMETRY

At a meeting of the Electors held on Friday, December 12, GODFREY HAROLD HARDY, MA, Fellow of Trinity College, Cambridge, was elected Savilian Professor of Geometry, to enter on office on January 19, 1920.

The Electors for the Oxford Chair were:

H Blakiston, *Vice-Chancellor*

Revd. W Spooner, *Warden of New College* (of 'Spoonerism' fame)

H H Turner, *Savilian Professor of Astronomy*

E B Elliott, *Waynflete Professor of Pure Mathematics*

M J M Hill, *Professor of Mathematics at University College, London*

Percy MacMahon, *former President of the London Mathematical Society*

C H Sampson, *Principal of Brasenose College*

Hardy had initially encouraged W H Young, independent inventor of the Lebesgue integral, to apply. Later, when Hardy himself decided to apply, he asked Young to withdraw his name, which he did.

Hardy's inaugural lecture on some famous problems of the theory of numbers was given in the University Observatory on Tuesday 18 May at 5pm; it was later published.

In his inaugural lecture Hardy discussed Waring's problem: *Every positive integer can be written as the sum of at most 4 perfect squares, 9 cubes or 19 fourth powers; does this continue for n th powers, and how many powers are needed?*

This was Hardy's first involvement with Waring's problem, and during his Oxford years he co-authored six papers with J E Littlewood on it.

SOME FAMOUS PROBLEMS

of the

THEORY OF NUMBERS

and in particular

Waring's Problem

An Inaugural Lecture delivered before the

University of Oxford

BY

G. H. HARDY, M.A., F.R.S.

*Fellow of New College
Savilian Professor of Geometry in the University of Oxford
and late Fellow of Trinity College, Cambridge*

OXFORD
AT THE CLARENDON PRESS
1920

G H Hardy's Oxford years

Hardy in Oxford



Mathematical
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Hardy found himself well suited to life at New College, the college attached to the Savilian chairs.

Hardy at New College

At New College the Wardens during Hardy's time were the Revd. William Spooner, who lectured on ancient history, philosophy and divinity, and (from 1925) H A L Fisher, who had served in government and later wrote a celebrated *History of Europe*.

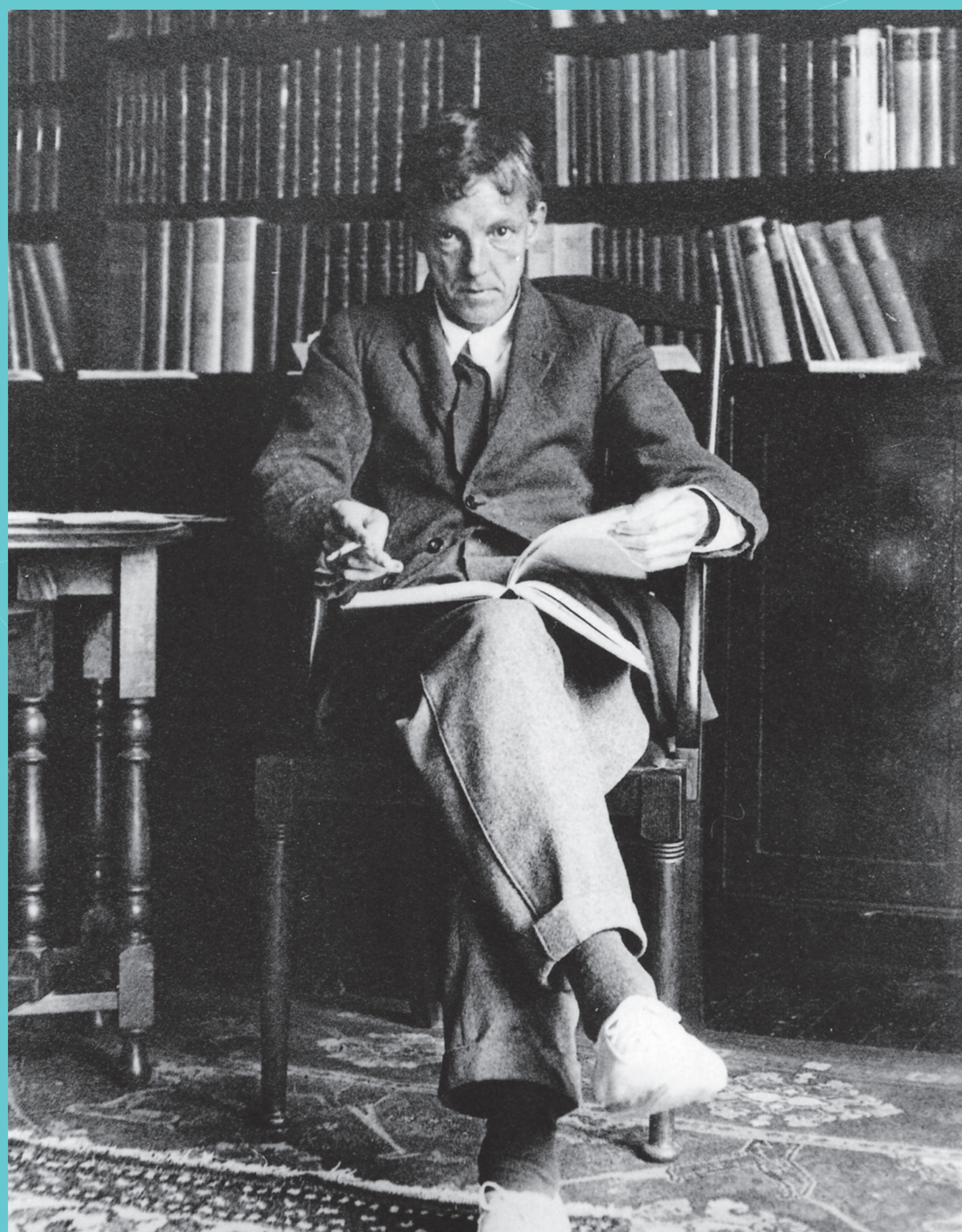
New College Fellows in the 1920s included J S Haldane, H W B Joseph and Hugh Allen. The College then had 300 students, but at most one student took mathematics Finals each year. Among the New College students during Hardy's time were Hugh Gaitskell, Richard Crossman and Lord Longford.

"In the informality and friendliness of New College Hardy always felt completely at home. He was an entertaining talker on a great variety of subjects, and one sometimes noticed everyone in common room waiting to see what he was going to talk about."

E C Titchmarsh
Hardy's successor as Savilian Professor

"... the leader of talk in the senior common room, arranging with mock seriousness complicated games and intelligence tests, his personality gave a quality to the atmosphere quite unlike anything else in my experience..."

Lionel Robbins
Economist



Hardy in his room at New College

Mathematical contemporaries

On the mathematical side, Hardy's Oxford colleagues initially consisted of the Sedleian Professor of Natural Philosophy and the Waynflete Professor of Pure Mathematics, the Savilian Professor of Astronomy, and a number of college tutors. Another mathematics chair was added in 1928.

The Applied mathematics professor was Augustus Love, who occupied the Sedleian Chair for over forty years. He was President of the London Mathematical Society and was awarded its De Morgan Medal. His monumental text on the mathematical theory of elasticity was of great importance, and his lectures to Oxford students were models of clear thinking and style.



A. E. H. Love

Twenty years earlier Love had taught Hardy at Cambridge, as Hardy recalled:

"My eyes were first opened by Professor Love, who taught me and gave me my first serious conception of analysis. But the great debt which I owe to him was his advice to read Jordan's famous *Cours d'Analyse*; and I shall never forget the astonishment with which I read that remarkable work, the first inspiration for so many mathematicians of my generation, and I learnt for the first time what mathematics really meant. From that time onwards I was in my way a real mathematician, with sound mathematical ambitions and a genuine passion for mathematics."

The Waynflete Chair of Pure Mathematics was created in 1892. Its first occupant was E B Elliott. His interests were in algebra and his textbook on invariant theory was highly regarded, but his mathematics was old-fashioned, mainly involving the exploitation of elementary ideas with skill and insight. He had 'no sympathy with foreign modern symbolic methods'.



E B Elliott

Elliott retired as Waynflete Professor in 1921 and was replaced by A L Dixon, who studied the applications of algebra to geometry and elliptic functions. Both Elliott and Dixon were Presidents of the London Mathematical Society.

In 1928 the Rouse Ball Chair was created in Oxford. Unlike its Cambridge counterpart, where it is reserved for pure mathematics, the Oxford Chair is always held by applied mathematicians.

Its first occupant was E A Milne, a former student of Hardy's in Cambridge. A major figure in astrophysics, Milne revolutionised applied mathematics at Oxford.

Later holders of the Oxford Rouse Ball Chair included C A Coulson and Sir Roger Penrose.

G H Hardy's Oxford years

Teaching and research



Mathematical
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While in Oxford Hardy distinguished himself as both a superb lecturer and an inspiring leader of research.

Hardy's teaching

Hardy was a brilliant and widely admired lecturer. One of his Oxford students later wrote: "Each lecture was carefully prepared, like a work of art, with the intellectual denouement appearing as if spontaneously in the last five minutes of the hour. For me these lectures were an intoxicating joy."

In common with most of his Oxford colleagues Hardy taught in his own college – there was then no Mathematical Institute.

Although not a geometer, Hardy assiduously lectured on geometry each term – *Analytic geometry*, *Applications of analysis to geometry*, *Solid geometry* and *Elements of non-Euclidean geometry* – as well as branching out into non-geometrical topics.

The Hilary Term lecture list for 1920 included a range of other lecture courses, including *Theory of functions* by Elliott, *Mathematical astronomy* by Turner, *Electricity and magnetism* by Love, and *Hydrostatics* by Dixon.

Halfway through the term Hardy commenced a course of lectures introducing the *Analytical geometry of the plane*, given on Tuesdays and Saturdays at 11am in New College.

Hardy's lecture courses were always models of good organization and clarity. Fortunately, several of them have been preserved in notes taken by his Oxford research student E H Linfoot. Below is a page from a number theory course given by Hardy in 1924–25.

Hardy's research

Before Hardy there was no flourishing research tradition in Oxford, although J J Sylvester had tried to initiate one in the 1880s and particular individuals such as Augustus Love were involved in their own researches. In 1925 E B Elliott remarked: "I still hold soundly that our business as teachers in a University was to educate, to assist young men to make the best use of their powers ... 'But how about research and original work under this famous system of yours? You do not seem to have promoted it much.' Perhaps not! It had not yet occurred to people that systematic training for it was possible."

Hardy determined to change all this. His own research interests were mainly in number theory and mathematical analysis, and his greatest legacy to Oxford mathematics was the internationally renowned research school of analysis that he established there in the 1920s.

Shortly after arriving in Oxford, he introduced Friday evening advanced classes in pure mathematics at New College, at which he, his research students, and any mathematical visitors to Oxford, discussed their current investigations.

The fact that Hardy and J E Littlewood were now at different universities made no difference to their productivity; during Hardy's Oxford years they produced about fifty joint papers. Probably the most prolific partnership in mathematical history, they co-authored almost one hundred papers on a wide range of subjects.



J E Littlewood

A joke of the time held that: *Nowadays, there are only three really great English mathematicians: Hardy, Littlewood and Hardy-Littlewood.*

"The mathematician Hardy-Littlewood was the best in the world, with Littlewood the more original genius and Hardy the better journalist."
Edmund Landau



Hardy's research students included E C Titchmarsh, Mary Cartwright (pictured above), E H Linfoot and his later co-author E M Wright.

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G H Hardy's Oxford years

Oxford preoccupations



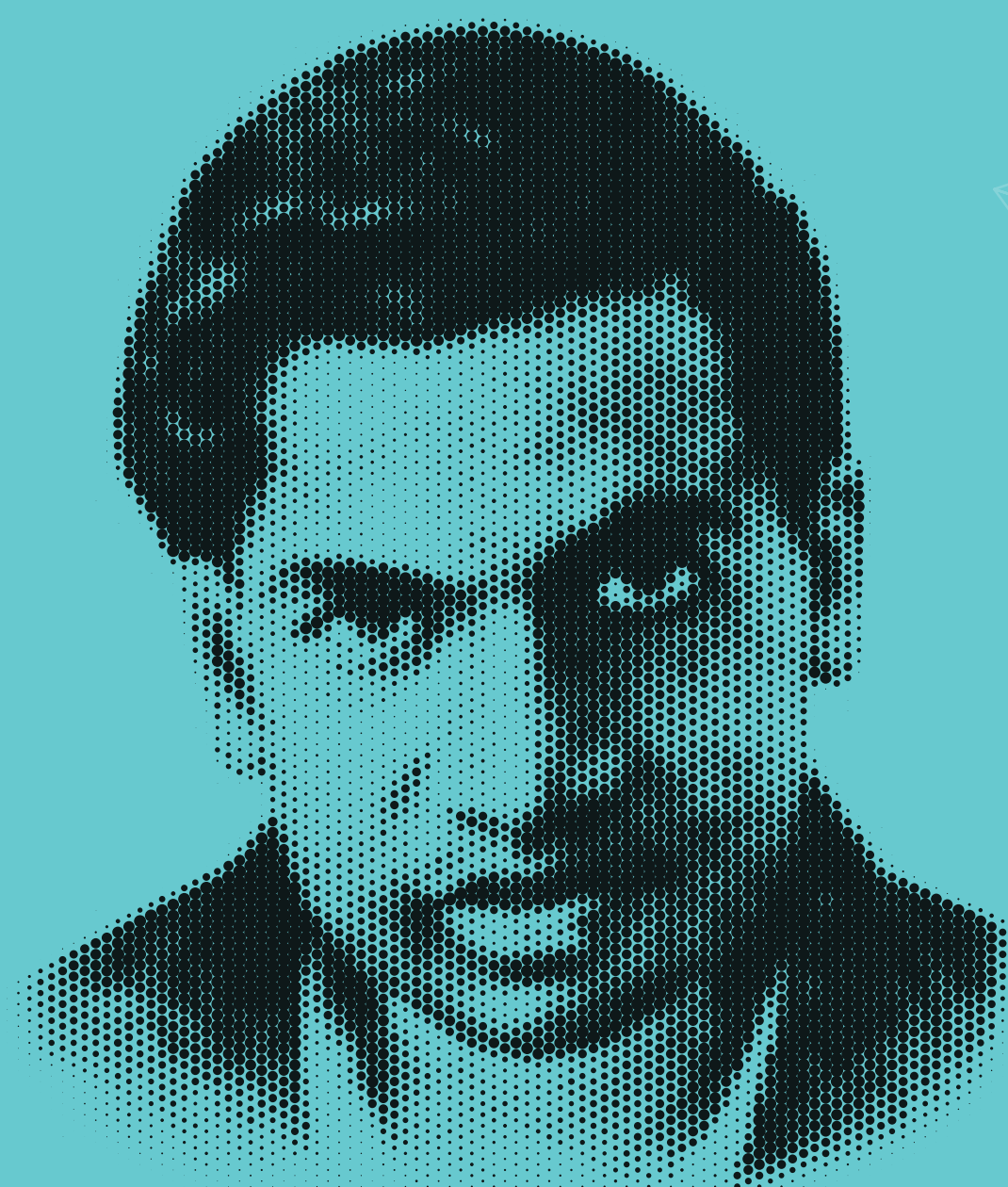
Mathematical Institute

In addition to his many research papers, Hardy was involved with a range of other activities.

Hardy's publishing activities

Hardy and Ramanujan

In the years prior to Hardy's coming to Oxford he produced a number of spectacular joint papers with the brilliant Indian mathematician Srinivasa Ramanujan. But Ramanujan became ill and returned to Madras in March 1919. He died a year later, shortly after Hardy's arrival in Oxford. Hardy was devastated: "For my part, it is difficult for me to say what I owe to Ramanujan – his originality has been a constant source of suggestion to me ever since I knew him, and his death is one of the worst blows I have ever had."



In 1921 Hardy wrote a beautifully crafted obituary notice, described by the *Manchester Guardian* as 'among the most remarkable in the literature about mathematics'. For the next three years Hardy wrote a number of papers based on, and extending, Ramanujan's work. Meanwhile, he was working through his colleague's extensive notebooks and papers, and in 1927, with two others, he published *Ramanujan's Collected Papers*.

Other Publications

In 1926 Hardy persuaded the London Mathematical Society to found the *Journal of the London Mathematical Society* and in 1930 he was involved in re-launching the *Quarterly Journal of Mathematics*. The first issue (shown below) featured many prominent Oxford mathematicians of the time.

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Hardy also found time for the occasional general article. In the *Oxford Magazine* of June 1930, he bemoaned the University's lack of attention to mathematics and encouraged it to build up a fine mathematical school, observing: 'mathematicians are reasonably cheap, but they cannot be had for nothing'. He urged an increase in the number of mathematical lecturers, proposed more research activity as essential for the future, and lobbied for the creation of a Mathematical Institute.

3. Ramanujan's proof of $p(11n+6) \equiv 0 \pmod{11}$ (11)
 (Formulae quoted from 'Some Arith. Thms', C.P.S.)

1. $QR = 1 + 11J$ (J generally = power series with integral coefficients)

$$Q^3 - 3R^3 = 44Q^3 + 150R^3 + 11J$$

$$= 691 + 65510 \left(\frac{1}{1-2} + \frac{2^{11}2^9}{1-2^{11}} + \dots \right) + 11J$$

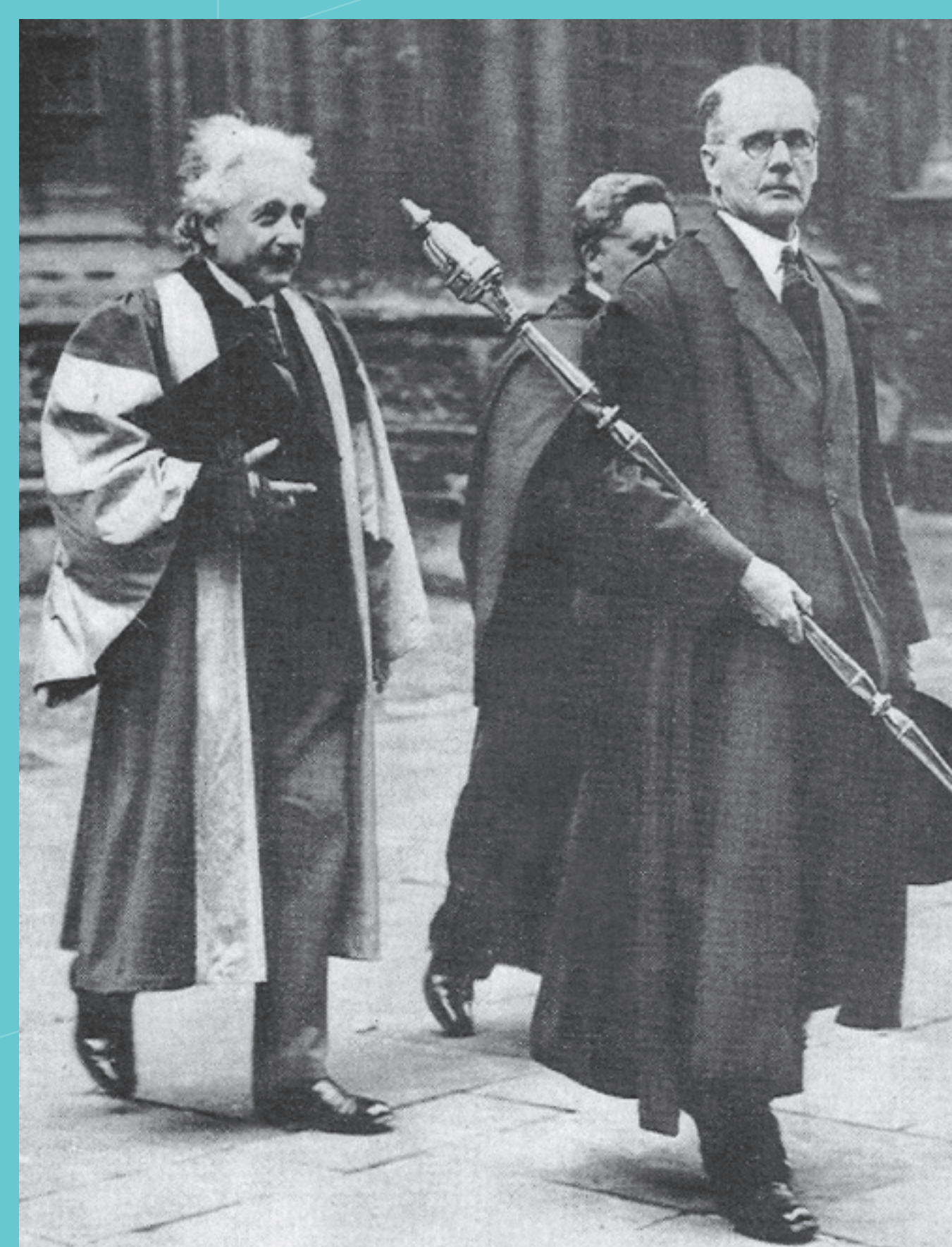
$$= -2 + 48 \left(\frac{1}{1-2} + \frac{2^9}{1-2^9} + \dots \right) + 11J$$

$$= -27 + 11J$$

Other Oxford preoccupations

Einstein visits Oxford

In May 1931 a major scientific event took place in Oxford when Albert Einstein received an honorary Doctorate of Science and presented three lectures on the theory of relativity at Rhodes House; a blackboard from one lecture is still preserved in Oxford. Merton College arranged a special dinner for Einstein, which Hardy, Love, Dixon and Milne attended.



Hardy's feud with God

'God-free' Harold Hardy was an ardent atheist who had a permanent feud with God.

Below is a bizarre note that he wrote in the form of an imaginary cricket match between himself and God, in which Hardy, of course, won.

New College, Oxford.

What a day!

	from	g
Deat Jan at 1/11/11	67	33
5-2		
Good meth. thm (O.K.)	73	0
211 in 1 hr at 1/11/11, making 33 (my Oxford score), 71		0
21 etc		
Villa 8-1	43	0
Anglo 5-1	31	0
Celtic match off	0	0
Nichol head (forget his name)	33	0
Nichol to be hanged	61	10
(EX)		
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Hardy's main relaxation was cricket. Here, during a British Association meeting in Oxford in August 1926, he leads a team of MATHEMATICIANS versus THE REST OF THE WORLD. His team included Titchmarsh, Bosanquet, Linfoot and Ferrar.

G H Hardy's Oxford years

From Oxford to Cambridge



Mathematical Institute

After eleven fruitful years in Oxford, Hardy decided to return to Cambridge for the rest of his days.

Hardy leaves Oxford

Although he was very happy at Oxford, Hardy chose to resign his Oxford Chair and return to Cambridge, following the death in 1931 of E W Hobson, Cambridge's Sadleirian Professor of Mathematics.

In spite of all Hardy's efforts to establish an analytical school in Oxford, Cambridge remained the mathematical hub of England and Hardy wished to be considered for its most prestigious mathematical chair. Moreover, he was now aged 54, and if he remained at New College he would have to yield up his rooms upon retirement; if he returned to Trinity College he could live there for life, as indeed did Littlewood.

Hardy was successful in securing the Sadleirian chair, and the following notice duly appeared in the *Oxford University Gazette*:

SAVILIAN PROFESSORSHIP OF GEOMETRY

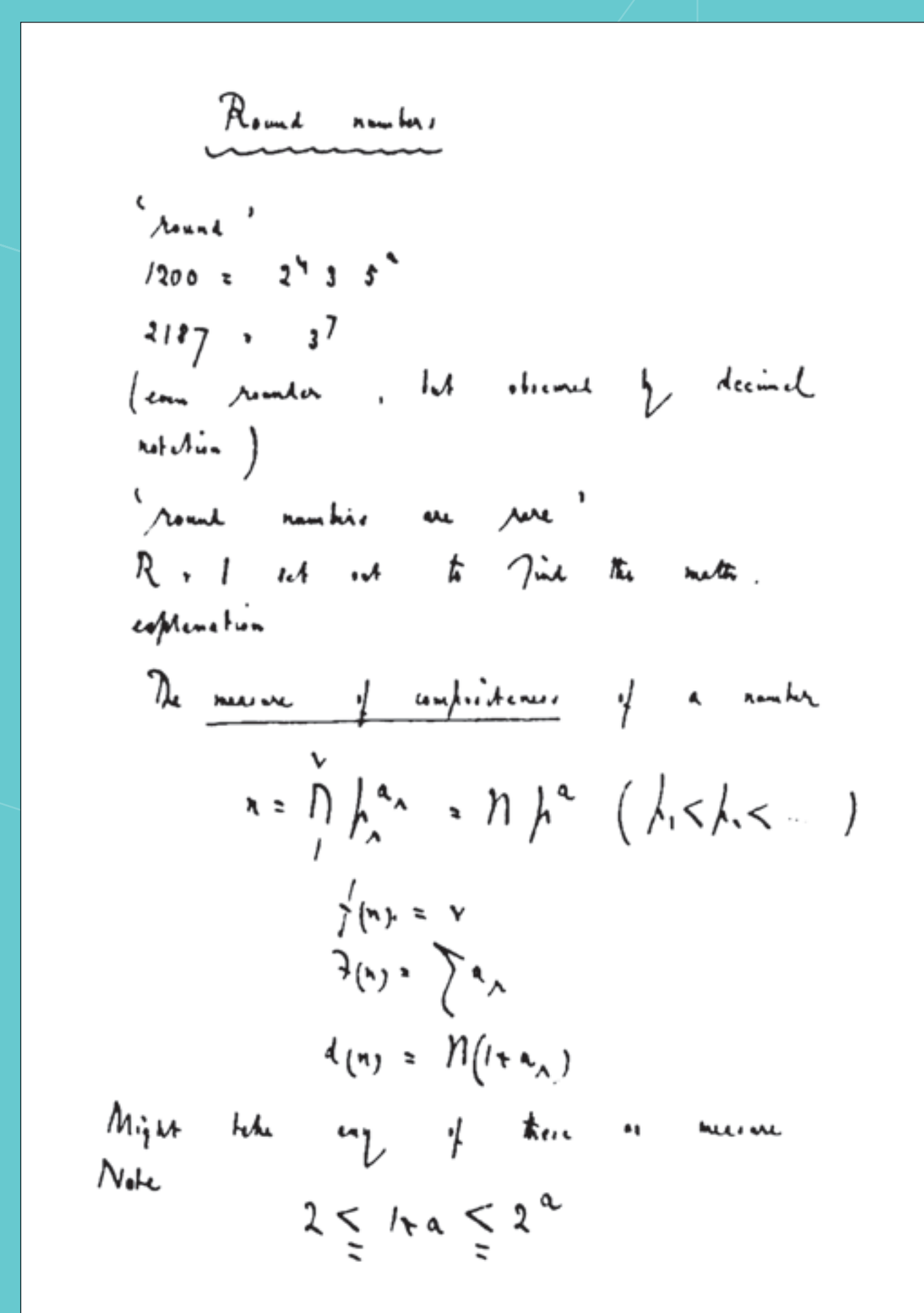
The Electors to this Professorship propose shortly to proceed to an election of a professor in the place of Godfrey Harold Hardy, MA, Fellow of New College, who has resigned as from the beginning of Michaelmas Term, 1931.

By this time the stipend had increased to £1200 per year, and the Statutes now recognised the importance of research: *The duties of every Professor shall include original work by the Professor himself and the general supervision of research and advanced work in his subject and department.*

Hardy's successor as Oxford's Savilian professor was his first research student, E C Titchmarsh, who explicitly asserted that he would not lecture in geometry. Since then, the specification for the Chair has been relaxed and its occupant can now be selected from any area of pure mathematics.

After his departure to Cambridge, Hardy returned to Oxford for several weeks every summer to enjoy the atmosphere, see his college and mathematical colleagues, and captain the New College Senior Common Room cricket team in their annual match against the college servants and the choir school.

He also returned to give the first lecture to the **Invariant Society**, Oxford University's undergraduate mathematics society (still flourishing); this was in Hilary Term 1936 on the subject of *Round numbers* (those with few different prime factors).



Sayings of G H Hardy

In the 1920s Hardy had many preoccupations besides his teaching and research. He was heavily involved with several national societies, becoming President of the Mathematical Association:

In his first presidential address, *What is geometry?*, Hardy asserted that: "I do not claim to know any geometry, but I do claim to understand quite clearly what geometry is."

He also joked that the only geometrical result that he had ever proved was: "If a rectangular hyperbola is a parabola, then it is also an equiangular spiral."



224. [M¹. 8. g.] A curious imaginary curve.
 The curve $(x + iy)^2 = \lambda(x - iy)$ is (i) a parabola, (ii) a rectangular hyperbola, and (iii) an equiangular spiral. The first two statements are evidently true. The polar equation is $r = \lambda e^{-3i\theta}$, the equation of an equiangular spiral. The intrinsic equation is easily found to be $\rho = 3is$. It is instructive (i) to show that the equation of any curve which is both a parabola and a rectangular hyperbola can be put in the form given above, or in the form $(x + iy)^2 = x$ (or y), and (ii) to determine the intrinsic equation directly from one of the latter forms of the Cartesian equation. **G. H. HARDY.**

As President of the National Union of Scientific Workers, Hardy's left-wing leanings emerge clearly from the following remarks to an audience of scientists: "...although our jobs are very different from a coalminer's we are much closer to coalminers than capitalists. At least we and the miners are both skilled workers, not exploiters of other people's work, and if there's going to be a line-up, I'm with the miners."

On visiting America: During his year in America Hardy wrote to a New College colleague: "This country is in many ways the only one in the world - it has its deficiencies (tea γ , paper β -, ... , dinner rather an uninteresting absolutely flat: the sun shines more or less continuously: and for quietness and the opportunity to be your own master I've never come across anything like it."

These posters were conceived by Robin Wilson, with the assistance of Raymond Flood and Dyrol Lombard.